Solutions: Questions 12–17

Q12. Find value of *p* so roots of x² + k(2x + k + 2) + p = 0 are equal

Step 1: Since 3 is a root of $x^2 - x + k = 0$:

$$9 - 3 + k = 0 \implies \boxed{k = -6}$$

Step 2: Substitute k = −6 in the second equation:

$$x^2 + (-6)(2x - 6 + 2) + p = 0$$

$$x^2 - 12x + 24 + p = 0$$

Step 3: For equal roots, Discriminant = 0:

$$(-12)^2 - 4(1)(24 + p) = 0$$

$$144 - 96 - 4p = 0 \implies \boxed{p = 12}$$

Q13. Find value of *k* so x² + px(1 + 3k) + 7(3 + 2k) = 0 has equal roots

Step 1: Since −4 is a root of $x^2 + 2x + 4p = 0$:

$$16 - 8 + 4p = 0 \implies \boxed{p = -2}$$

Step 2: Substitute p = −2:

$$x^2 - 2(1 + 3k)x + 7(3 + 2k) = 0$$

Step 3: For equal roots, D = 0:

$$4(1 + 3k)^2 - 28(3 + 2k) = 0$$

$$9k^2 - 8k - 20 = 0$$

$$(9k + 10)(k - 2) = 0$$

$$\boxed{k = 2 \quad \text{or} \quad k = -\frac{10}{9}}$$

Q14. Prove c² = a²(1 + m²) for equal roots of (1 + m²)x² + 2mcx + c² − a² = 0

For equal roots, D = 0:

$$(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$$

$$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$$

$$-4c^2 + 4a^2(1 + m^2) = 0$$

$$\boxed{c^2 = a^2(1 + m^2)} \qquad \textbf{Hence proved}\checkmark$$

Q15. Show a = 0 OR a³ + b³ + c³ = 3abc

For $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ with equal roots, D = 0:

$$4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

Expanding:

$$a^4 - 2a^2bc + b^2c^2 - b^2c^2 + ac^3 + ab^3 - a^2bc = 0$$

$$a^4 + ab^3 + ac^3 - 3a^2bc = 0$$

$$a(a^3 + b^3 + c^3 - 3abc) = 0$$

$$\boxed{a = 0 \quad \text{or} \quad a^3 + b^3 + c^3 = 3abc} \qquad \textbf{Hence proved}\checkmark$$

Q16. Find values of *p* for which 2x² + px + 8 = 0 has real roots

For real roots, D ⩾ 0:

$$p^2 - 4(2)(8) \geq 0$$

$$p^2 \geq 64$$

$$\boxed{p \leq -8 \quad \text{or} \quad p \geq 8}$$

Q17. Find α for which (α − 12)x² + 2(α − 12)x + 2 = 0 has equal roots

> **Note:** If α = 12, the equation becomes 0 = 2 (impossible). So α ≠ 12.

For equal roots, D = 0:

$$[2(\alpha - 12)]^2 - 4(\alpha - 12)(2) = 0$$

$$4(\alpha - 12)[(\alpha - 12) - 2] = 0$$

$$(\alpha - 12)(\alpha - 14) = 0$$

Since α ≠ 12:

$$\boxed{\alpha = 14}$$

Give full solution

Complete Detailed Solutions: Q12 to Q17

Q12. If 3 is a root of x² − x + k = 0, find p so that roots of x² + k(2x + k + 2) + p = 0 are equal.

Step 1: Find k using x = 3 in first equation

$$x^2 - x + k = 0$$

Substituting x = 3:

$$3^2 - 3 + k = 0$$

$$9 - 3 + k = 0$$

$$6 + k = 0$$

$$\boxed{k = -6}$$

Step 2: Substitute k = −6 in second equation

$$x^2 + k(2x + k + 2) + p = 0$$

$$x^2 + (-6)(2x + (-6) + 2) + p = 0$$

$$x^2 + (-6)(2x - 4) + p = 0$$

$$x^2 - 12x + 24 + p = 0$$

Step 3: Apply condition for equal roots → D = 0

Here: a = 1, b = −12, c = 24 + p

$$D = b^2 - 4ac = 0$$

$$(-12)^2 - 4(1)(24 + p) = 0$$

$$144 - 96 - 4p = 0$$

$$48 - 4p = 0$$

$$4p = 48$$

$$\boxed{p = 12}$$

Q13. If −4 is a root of x² + 2x + 4p = 0, find k so that x² + px(1 + 3k) + 7(3 + 2k) = 0 has equal roots.

Step 1: Find p using x = −4 in first equation

$$x^2 + 2x + 4p = 0$$

Substituting x = −4:

$$(-4)^2 + 2(-4) + 4p = 0$$

$$16 - 8 + 4p = 0$$

$$8 + 4p = 0$$

$$4p = -8$$

$$\boxed{p = -2}$$

Step 2: Substitute p = −2 in second equation

$$x^2 + px(1 + 3k) + 7(3 + 2k) = 0$$

$$x^2 + (-2)(1 + 3k)x + 7(3 + 2k) = 0$$

$$x^2 - 2(1 + 3k)x + 7(3 + 2k) = 0$$

Step 3: Apply D = 0 for equal roots

Here: a = 1, b = −2(1 + 3k), c = 7(3 + 2k)

$$D = b^2 - 4ac = 0$$

$$[-2(1 + 3k)]^2 - 4(1)[7(3 + 2k)] = 0$$

$$4(1 + 3k)^2 - 28(3 + 2k) = 0$$

Step 4: Expand and simplify

$$4(1 + 6k + 9k^2) - 84 - 56k = 0$$

$$4 + 24k + 36k^2 - 84 - 56k = 0$$

$$36k^2 - 32k - 80 = 0$$

Dividing everything by 4:

$$9k^2 - 8k - 20 = 0$$

Step 5: Solve by factorisation

$$9k^2 - 18k + 10k - 20 = 0$$

$$9k(k - 2) + 10(k - 2) = 0$$

$$(9k + 10)(k - 2) = 0$$

$$\boxed{k = 2 \quad \text{or} \quad k = -\frac{10}{9}}$$

Q14. If (1 + m²)x² + 2mcx + c² − a² = 0 has equal roots, prove that c² = a²(1 + m²)

Given equation: $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$

Here: $A = (1 + m^2)$, $B = 2mc$, $C = (c^2 - a^2)$

Condition for equal roots → D = 0:

$$D = B^2 - 4AC = 0$$

$$(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$$

$$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$$

Divide throughout by 4:

$$m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$$

The m²c² terms cancel:

$$-c^2 + a^2 + m^2a^2 = 0$$

$$a^2 + m^2a^2 = c^2$$

$$a^2(1 + m^2) = c^2$$

$$\boxed{c^2 = a^2(1 + m^2)} \qquad \textbf{Hence Proved} \checkmark$$

Q15. If roots of (c²−ab)x² − 2(a²−bc)x + (b²−ac) = 0 are real and equal, show a = 0 OR a³+b³+c³ = 3abc

Given equation: $(c^2-ab)x^2 - 2(a^2-bc)x + (b^2-ac) = 0$

Here: $A = (c^2-ab)$, $B = -2(a^2-bc)$, $C = (b^2-ac)$

Condition for equal real roots → D = 0:

$$B^2 - 4AC = 0$$

$$[-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

$$4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

Divide by 4:

$$(a^2 - bc)^2 - (c^2 - ab)(b^2 - ac) = 0$$

Expand (a²−bc)²:

$$= a^4 - 2a^2bc + b^2c^2$$

Expand (c²−ab)(b²−ac):

$$= c^2b^2 - ac^3 - ab^3 + a^2bc$$

Substituting back:

$$a^4 - 2a^2bc + b^2c^2 - (b^2c^2 - ac^3 - ab^3 + a^2bc) = 0$$

$$a^4 - 2a^2bc + b^2c^2 - b^2c^2 + ac^3 + ab^3 - a^2bc = 0$$

$$a^4 - 3a^2bc + ab^3 + ac^3 = 0$$

$$a(a^3 - 3abc + b^3 + c^3) = 0$$

$$a(a^3 + b^3 + c^3 - 3abc) = 0$$

Either:

$$a = 0$$

OR

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$a^3 + b^3 + c^3 = 3abc$$

$$\boxed{a = 0 \quad \text{OR} \quad a^3 + b^3 + c^3 = 3abc} \qquad \textbf{Hence Proved} \checkmark$$

Q16. Find values of p for which 2x² + px + 8 = 0 has real roots.

For real roots → D ⩾ 0

Here: a = 2, b = p, c = 8

$$D = b^2 - 4ac \geq 0$$

$$p^2 - 4(2)(8) \geq 0$$

$$p^2 - 64 \geq 0$$

$$p^2 \geq 64$$

$$|p| \geq 8$$

$$\boxed{p \leq -8 \quad \text{or} \quad p \geq 8}$$

Q17. Find α for which (α−12)x² + 2(α−12)x + 2 = 0 has equal roots.

Note: If α = 12, equation becomes 0·x² + 0·x + 2 = 0 → 2 = 0 (impossible) ∴ **α ≠ 12**

Condition for equal roots → D = 0

Here: a = (α−12), b = 2(α−12), c = 2

$$D = b^2 - 4ac = 0$$

$$[2(\alpha - 12)]^2 - 4(\alpha - 12)(2) = 0$$

$$4(\alpha - 12)^2 - 8(\alpha - 12) = 0$$

Take 4(α−12) common:

$$4(\alpha - 12)\left[(\alpha - 12) - 2\right] = 0$$

$$4(\alpha - 12)(\alpha - 14) = 0$$

Either:

$$\alpha - 12 = 0 \Rightarrow \alpha = 12 \quad \text{(Rejected, as shown above)}$$

$$\alpha - 14 = 0 \Rightarrow \alpha = 14 \quad \checkmark$$

$$\boxed{\alpha = 14}$$

Verification: Put α = 14: $(14 - 12)x^2 + 2(14 - 12)x + 2 = 0$ $2x^2 + 4x + 2 = 0$ $x^2 + 2x + 1 = 0$ $(x + 1)^2 = 0$ → x = −1, −1 ✓ *(equal roots confirmed)*